QUIMICA FISICA II. CURSO 2003-2004.
Expresiones e integrales útiles

$$\int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}} \tag{1}$$

$$\int_x^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}} e^{-\alpha x} \sum_{i=0}^{n} \frac{(\alpha x)^i}{i!} \tag{2}$$

$$\int_0^\infty x^{2n+1} e^{-\alpha x^2} dx = \frac{n!}{2\alpha^{n+1}} \tag{3}$$

$$\int_x^\infty x^{2n+1} e^{-\alpha x^2} dx = \frac{n!}{2\alpha^{n+1}} e^{-\alpha x^2} \sum_{i=0}^{n} \frac{(\alpha x^2)^i}{i!} \tag{4}$$

$$\int_0^\infty x^{2n} e^{-\alpha x^2} dx = \frac{(2n-1)!!}{2^{n+1}\alpha^n} \sqrt{\frac{\pi}{\alpha}} \tag{5}$$

$$\int_x^\infty x^{2n} e^{-\alpha x^2} dx = \frac{(2n-1)!!}{2^{n+1}\alpha^n} \left[\frac{e^{-\alpha x^2}}{\alpha x} \sum_{i=1}^{n} \frac{(2\alpha x^2)^i}{(2i-1)!!} + \sqrt{\frac{\pi}{\alpha}} F_c(\sqrt{\alpha} x) \right] \tag{6}$$

donde $F_c(x) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_x^\infty e^{-t^2} dt$, con lo que $F_c(0) = 1$

$$\int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2} \left(\frac{\pi}{\alpha} \right)^{1/2}, \qquad \int_0^\infty x e^{-\alpha x^2} dx = \frac{1}{2\alpha}, \tag{7}$$

$$\int_0^\infty x^2 e^{-\alpha x^2} dx = \frac{1}{4\alpha} \left(\frac{\pi}{\alpha} \right)^{1/2}, \qquad \int_0^\infty x^3 e^{-\alpha x^2} dx = \frac{1}{2\alpha^2}, \tag{8}$$

$$\int_0^\infty x^4 e^{-\alpha x^2} dx = \frac{3}{8\alpha^2} \left(\frac{\pi}{\alpha} \right)^{1/2}, \qquad \int_0^\infty x^5 e^{-\alpha x^2} dx = \frac{1}{\alpha^3}, \tag{9}$$

$$\int_0^\infty x^6 e^{-\alpha x^2} dx = \frac{15}{16\alpha^3} \left(\frac{\pi}{\alpha} \right)^{1/2}, \qquad \int_0^\infty x^7 e^{-\alpha x^2} dx = \frac{3}{\alpha^4}, \tag{10}$$

$$\int_0^\infty x^8 e^{-\alpha x^2} dx = \frac{105}{32\alpha^4} \left(\frac{\pi}{\alpha} \right)^{1/2}, \qquad \int_0^\infty x^9 e^{-\alpha x^2} dx = \frac{12}{\alpha^5}. \tag{11}$$

En estas expresiones $n!! = n(n-2)(n-4)...$ es el equivalente a un factorial en el que sólo se multiplican los números de la misma paridad que n. Así: $6!! = 6 \cdot 4 \cdot 2 = 48$, y $5!! = 5 \cdot 3 \cdot 1 = 15$.